Algorithm of the Risk of Ship-Bridge Collision Considering Ship's Dimension

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Abstract

The issue of ship collision with bridge is very important topic in the academic research, however, the studies only regard the ship as a particle at present, ignoring the size of the ship, which cause the accuracy of the active early warning system cannot meet the requirements of practice. In this paper, proposes the algorithm of the risk of ship-bridge collision considering the ship's dimension, including the L_{OA} and B_{MAX}. First, utilizing the mercator projection method to make convertion of coordinate system. Secondly, analysing the relationship of geometric position based on the ships, bridge piers and channel to constructe the algorithm of the risk ship collision with bridge. Thirdly, the active early warning strategy is proposed depending on the value from the descripted algorithm. Finally, the reliability of the algorithm is validated by case study. It can improve the accuracy of the risk of ship-bridge collision significantly, and conducive to the application of the warning system.

Keywords: ship collision with bridge; ship's dimensions; position coordinates; risk degree; channel centerline

1 Introduction

With the development of economy and water transportation, the accidents of ship-bridge collision is increasing[1], which makes the navigational environment complicated sharply, and the potential risk of ship collision with bridge is concerned widely by the industry. In order to deal with the adverse effects of uncertain factors such as human factor in the accidents, an active early warning system for ship-bridge collision avoidance was born at the right moment. The key to the function of the system depending on the core index of the risk of ship collision with bridge which touching off the facilities and equipment for warning.The algorithm of the risk of ship-bridge collision and the strategy of early warning are the core key of the system.It is the critical key that the system can be applied to practice really.

Over the years, domestic and foreign scholars have studied the algorithm of ship collision risk from different aspects. Through literature research, the author found that: (1) The traditional methods for anti-collision for bridge only focused on itself, it just depend on its' strong structure to against ship collision in order to minimize the loss. For example, Keke Peng[2] adopted AASHTO method to calculate the ship-bridge collision probability from the perspective of bridge; Tao Fu[3] studied ship collision with bridge based on structural reliability theory; (2) Most of the research adopted subjective qualitative analysis method at present, and only a few scholars studied it from an objective perspective. For example, Yihua Liu[4] designed an algorithm to describe the risk of ship-bridge collision from the perspective of the distance and relative speed of two ships; Jun Zhong[5] combined AHP method and entropy weight method to calculate the weight of each index affecting the risk of ship collision with bridge; (3) there are two situations that considering the ship factors in the study of the risk of ship-bridge collision: first, the AIS position of the ship is regarded as the ship position, regardless of the ship's length and width. Such as Wu Bing[6] and Yihua Liu[7]; Second, the Ship's dimension is blurred. Such as Changhai Huang[8], Xiangrui Yang[9], Xue Hui Yang[10], Jie Wang[11]. They considered the impact of ship's dimension on the risk of ship-bridge collision, but only give it an impact factor, which possess definite subjectivity.

The above research has promoted the development of the ship-bridge collision avoidance research, and there is a huge progress in the active early warning system for ship-bridge collision avoidance research. At present, some early-generation of the early warning system for ship-bridge collision avoidancehave appeared, but in practical application, missing alarm and false alarm often occurs. The reason may be that the algorithm is not accurate enough, so that the reliability of the early warning system is poor. In this paper, an algorithm is designed from the perspective of ship's dimension to calculate the risk of ship collision with bridge, improving the accuracy of the algorithm in the active early warning system for ship-bridge collision avoidance, and promoting the application of the early warning system in navigation.

2 Constructing algorithm

Firstly, the plane rectangular coordinate system is established according to the channel structure of the bridge, and utilizing the mercator projection method to convert the geographical location of the bridge and ship. Then the coordinates of ship's projection point E on the bridge axis along the channel direction is designed based on the relationship among ship, bridge and channel centerline. Finally, the risk algorithm of ship-bridge collision is constructed by using the distance between the projection point E and the intersection point K of the channel centerline and the bridge.

Based on the high-precision risk of ship-bridge collision, proposing the active warning strategy of ship-bridge collision avoidance. Through the warning strategy, the precision, reliability and practicability of the warning system are improved, and the common safety of bridge and ship is guaranteed. As shown in figure 1, it is the flow chart of the risk algorithm of the ship-bridge collision, where A is the position of pier close to the ship, EA is the distance between points E and A, and EK is the distance between points E and K.

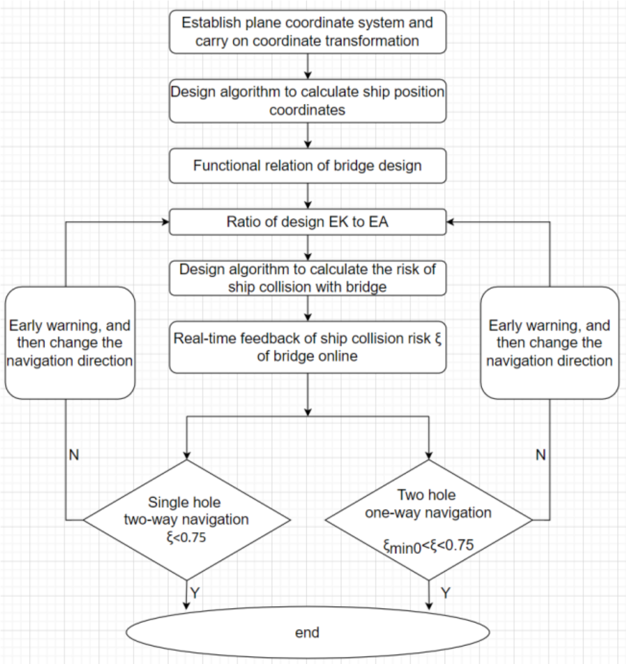

Figure 1 Flow chart of ship bridge collision risk algorithm and early warning

2.1 Establishment of rectangular coordinate system

As shown in figure 2, The channel entrance direction is taken as the positive direction of the longitudinal axis of the bridge-ship coordinate system. Since there are many channel entrance directions and there is only one channel centerline in the bridge area waterway, the channel centerline in the bridge area waterway is taken as the longitudinal axis (x-axis). The active early warning system's entrance direction is taken as the positive direction of Y axis and its' outer boundary GH in the entrance direction as the horizontal axis

(Y-axis). The intersection of the channel centerline in the bridge area waterway and the outer boundary GH is taken as the origin O (0,0). Point K is the intersection of X axis and Bridge AB.

The division of the active early warning system for ship-bridge collision avoidance: point S on X-axis is selected to make OK=KS, going through S and drawing a line JF parallel to Y-axis. Crossing pier A and drawing a straight line perpendicular to Y-axis, intersecting JF at point F and intersecting Y-axis at point H. Similarly, crossing pier B and drawing a straight line perpendicular to Y-axis, intersecting JF at point J and intersecting Y-axis at point G. i.e. JFHG in FIG. 2 is the active early warning range for ship-bridge collision avoidance.

When the ship sails into the outer boundary GH, the active early warning system for ship-bridge collision avoidance begins to work.

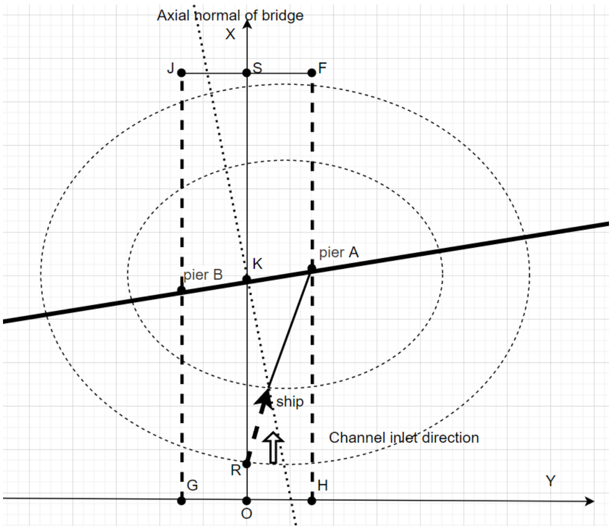

Figure 2 Bridge-ship coordinate system

2.2 Coordinate transformation

Utilizing mercator projection method, the longitude and latitude coordinate points (lat, lon) of ships and bridges are transformed into new coordinate points (x, y) in the bridge ship coordinate system. Since the earth is an ellipsoid, the semi-major axis of the ellipsoid is set as a, the short half axis of the ellipsoid is set as b, lat_0 is the longitude of the origin of the bridge-ship coordinate system, lon_0 is the latitude of the origin of the bridge-ship coordinate system, e is the first eccentricity of the ellipsoid, e' is the second eccentricity of the ellipsoid, and U and q are intermediate variables. Therefore, the coordinate conversion formula[12] is equation (1-4):

$$q = \ln\left[\tan\left(\frac{\pi}{4}+\frac{lat}{2}\right)\cdot\left(\frac{1-e\cdot\sin(lat)}{1+e\cdot\sin(lat)}\right)^{\frac{e}{2}}\right] \tag{1}$$

$$U = \frac{a^2/b}{\sqrt{1+e'^2\cos^2(lat_0)}} * \cos(lat_0) \tag{2}$$

$$X = Uq \tag{3}$$

$$Y = U(lon - lon_0) \tag{4}$$

2.3 Position coordinate projection position

As shown in figure 3, L is the ship length, Z is the ship width, and the position sensor on the ship is point C. The ship's course over ground (COG) is the angle θ between due north and the heading direction of a ship (i.e. the ship's sailing direction), α is the angle between the due north and the X axis (θ and α not shown in the figure), β is the angle between the heading direction of the ship and the X-axis (i.e. $\beta = \theta - \alpha$), D_1 is the left posterior position point of the ship (the leftmost point), and D_2 is the right anterior position point of the ship (the rightmost point).

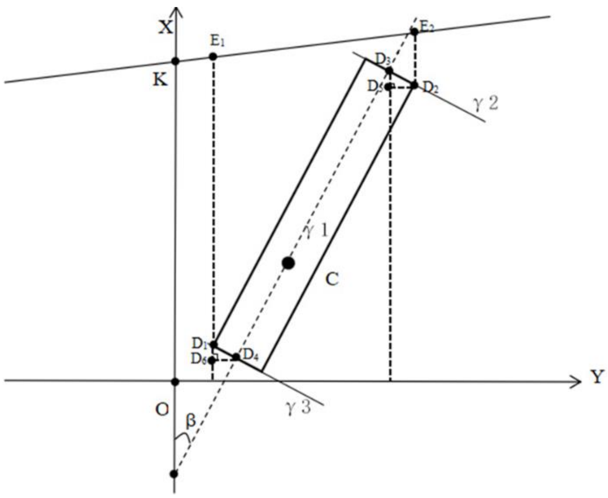

Figure 3 Projection of the ship on the bridge

In figure 3, γ_1 is the parallel line about the ship navigation direction passing through point C of the position sensor, γ_2 is the vertical line about the ship navigation direction passing through the ship head, γ_3 is the vertical line passing through the stern about ship navigation direction, D_3 is the intersection of γ_1 and γ_2, D_4 is the intersection of γ_1 and γ_3, Points E_1 and E_2 are the projection points of points D_1 and D_2 of the ship on bridge AB (point A and point B are the location points of the two piers of the bridge, as shown in Figure 2) (that is, E_1 and E_2 are the farthest and nearest projection points of the ship on bridge AB).

It can be seen from figure 3 that the coordinates (y_c, x_c) are the coordinates of point C where the position sensor on the ship, and the relationship between the coordinates (x, y) of D_i (i = 3,4) is equation (5):

$$X = |Y - y_c| \tan(90^\circ - |\beta|) + x_c \tag{5}$$

It is known that the captain is L and the ship breadth is Z. According to the position of ship position sensor C, setting $D_3C=0.68L$, $D_2D_3=0.51Z$, $D_4C= 0.32L$, $D_1D_4=0.49Z$.

The position coordinate of D_3 is set as (y_3, x_3), there are equation (6):

$$\begin{cases} 0.68L = \sqrt{(y_3 - y_c)^2 + (x_3 - x_c)^2} \\ x_3 = (y_3 - y_c)\tan(90^\circ - |\beta|) + x_c \end{cases} \tag{6}$$

The position coordinates (y_3, x_3) of D_3 is calculated from formula (6), and two coordinate points can be obtained. The position coordinates where y_3 is greater than y_c is taken as the position coordinates of D_3;

The position coordinates of D_4 is set as (y_4, x_4), there are equation (7):

$$\begin{cases} 0.32L = \sqrt{(y_4 - y_c)^2 + (x_4 - x_c)^2} \\ x_4 = (y_4 - y_c)\tan(90^\circ - |\beta|) + x_c \end{cases} \tag{7}$$

The position coordinates (y_4, x_4) of D_4 is calculated from equation (7), and two coordinate points can be obtained. The position coordinates where y_4 is less than y_c is taken as the position coordinates of D_4.

According to figure 3, in triangle $D_3D_2D_5$, the length of D_2D_3 is known to be 0.51Z, and according to the geometric relationship, $\angle D_3D_2D_5=\beta$,The horizontal scale of D_2 can be expressed as equation (8):

$$y_2 = y_3 + 0.51Z\sin(90^\circ - |\beta|) \tag{8}$$

Similarly, it can be concluded that in triangle $D_1D_4D_6$, the horizontal scale of D_1 can be expressed as equation (9):

$$y_1 = y_4 - 0.49Z\sin(90^\circ - |\beta|) \tag{9}$$

Since E_1 and E_2 are the projection points of D_1 and D_2 on the bridge respectively, the horizontal scale of E_1 and E_2 are y_1 and y_2.

From pier A(y_a,x_a) and pier B(y_b,x_b), the functional relationship of the bridge can be obtained as equation (10):

$$X = \frac{(Y - y_b)(x_a - x_b)}{y_a - y_b} + x_b \tag{10}$$

Let the coordinate of point K be (0, x_k), then x_k is equation (11):

$$x_K = x_b - \frac{y_b(x_a - x_b)}{y_a - y_b} \tag{11}$$

According to the horizontal scale of E_1 and E_2, the vertical coordinates of E_1 and E_2 are x_1 and x_2, that is, the coordinates of E_1 are (y_1, x_1) and the coordinates of E_2 are (y_2, x_2).

2.4 Construction of ship-bridge collision risk algorithm

As shown in figure 2, the active early warning system for ship-bridge collision aviodance is divided into four areas (equation 12), where x and y are the vertical and horizontal coordinate points of any point in the bridge-ship coordinate system: entrance normal navigation area OKAH (quadrant I), entrance reverse navigation area OKBG (quadrant II), exit normal navigation area SKBJ (quadrant III) and exit reverse navigation area SKAF (quadrant IV).

$$\begin{cases} x_{i+1} - x_i > 0, y_{i+1} > 0, quadrantI \\ x_{i+1} - x_i > 0, y_{i+1} < 0, quadrantII \\ x_{i+1} - x_i < 0, y_{i+1} < 0, quadrantIII \\ x_{i+1} - x_i < 0, y_{i+1} > 0, quadrantIV \end{cases} \tag{12}$$

Based on the coordinates of D_1, D_2, D_3 and D_4 and equation (12), judging whether the current position of the ship is within the range of the above four quadrants, and then judging whether the ship is in the normal navigation area (safe) or in the reverse navigation area (unsafe) according to the navigation direction of the ship. Further determine the quadrant where the ship is sailing. Sending intrusion warning to ships sailing outside the scope of active anti-collision warning, and sending reverse warning to ships sailing in the quadrant of reverse channel.

As shown in figure 2, the center of pier A is connected to the center of the ship, intersecting the channel centerline (X-axis) at point R. The center of pier A is connected to point R. Pier A is taken as the center of the circle, and the euclidean distance D_i between the ship and pier A is taken as the radius to draw a circle. Pier A is taken as the center of the circle and the distance D_i' between the center of pier A and point R is taken as the radius to draw a circle.

For all ships in the normal navigation area within the active early warning range of active anti-ship collision avoidance, the euclidean distance between each ship and the nearest Pier (This paper takes pier A as an example) is calculated. The distance E_1k and E_2k is calculated from E_1 and E_2 point to K point on the channel centerline and the distance E_1A and E_2A is calculated from pier A to E_1 and E_2 point. According to $E_1(y_1,x_1)$, $E_2(y_2,x_2)$, $K(0,x_K)$, $A(y_a,x_a)$, the length of E_1k,E_2K,E_1A and E_2A are equation (13-16):

$$E_1K=\sqrt{(y_1)^2+(x_1-x_K)^2} \tag{13}$$

$$E_2K=\sqrt{(y_2)^2+(x_2-x_K)^2} \tag{14}$$

$$E_1A=\sqrt{(y_a-y_1)^2+(x_a-x_1)^2} \tag{15}$$

$$E_2A=\sqrt{(y_a-y_2)^2+(x_a-x_2)^2} \tag{16}$$

According to the geometric relationship between bridge, pier and channel centerline, it can be known that the ratio of AK and E_iA is equal to the ratio of D_i' to D_i (i =1,2). According to D_i and D_i', the risk index model of ship collision with bridge is constructed, and the formula is equation (17-18):

$$\rho_1=\frac{D_i'-D_i}{D_i}=\frac{D_i'}{D_i}-1=\frac{E_1K}{E_1A} \tag{17}$$

$$\rho_2=\frac{D_i'-D_i}{D_i}=\frac{D_i'}{D_i}-1=\frac{E_2K}{E_2A} \tag{18}$$

According to formula 17, the ratio of E_1K to E_1A is ρ_1. The ship-bridge collision risk of the lower left endpoint of the ship D_1 is the lowest (because pier A is set as the closest pier to the ship in this paper), and the ship collision risk at point D_1 is set as ξ_1. According to formula 18, the ratio of E_2K to E_2A is ρ_2. The ship-bridge collision risk of the upper right endpoint D_2 is the largest. The ship collision risk of D_2 point is set as ξ_2, (e is a natural constant), ξ_1, ξ_2 is equation (19-20):

$$\xi_1=2.3\times10^{-3.3}e^{6.76\rho_1}\{0\le\rho_1\le1\} \tag{19}$$

$$\xi_2=2.3\times10^{-3.3}e^{6.76\rho_2}\{0\le\rho_2\le1\} \tag{20}$$

The risk of ship collision with the bridge is between $[\xi_1,\xi_2]$.

2.5 Active early warning strategy

As shown in figure 4, it is a graph of the risk of ship-bridge collision made by python. According to the tangent of the graph, it can be known that the risk change rate of the ship-bridge collision is large after the risk is 0.75. Therefore, the threshold of the risk early warning of ship-bridge collision involved in this paper is set to 0.75.

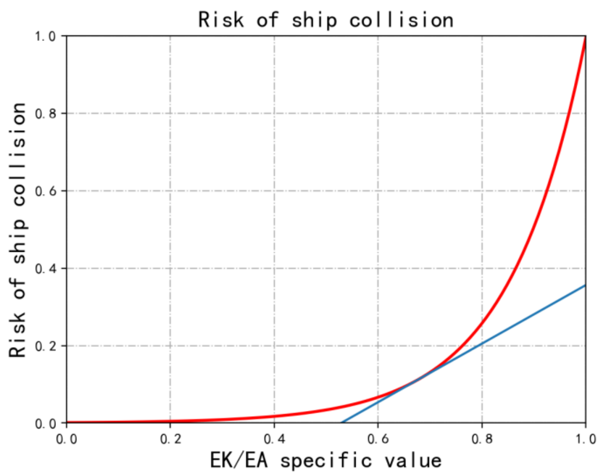

Figure 4 Danger curve of ship collision with bridge

The early warning function will be started under the following two conditions:

(1) The ship is in single hole two-way span of bridge. If the upper limit of the risk interval of collision of the ship ξ_2 is greater than or equal to the threshold of 0.75, the warning function is started.

The ship navigates in two holes one-way span of bridge. As shown in figure 5, there is a pier IMPN between piers A and B. It is necessary to consider the projection of the pier on the bridge along the channel direction. The projection point closest to pier A is set as E_3 and the furthest projection point is set as E_4. The ship navigation direction shown in figure 3 is taken as an example, when the projection point at the leftmost of the ship coincides with E_3, the risk of ship-bridge collision is ξ_{Min0}, therefore, the minimum risk of ship-bridge collision is ξ_{min0}. When the upper risk limit of the ship-bridge collision ξ_2 is greater than or equal to the threshold of 0.75, or the lower risk limit of the ship-bridge collision ξ_1 less than or equal to ξ_{min0}, starting the warning function.

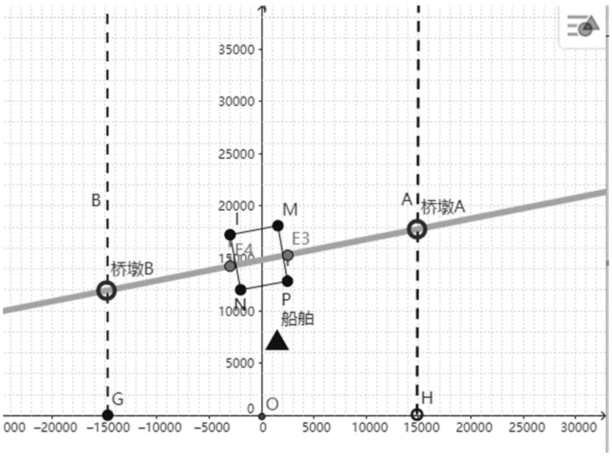

Figure 5 IMPN diagram of bridge piers between AB

(2) In a certain period of time(t_1,t_2): at time t_1, the minimum risk of ship-bridge collision i is ξ_{it_1}, and at time t_2, the maximum risk of ship-bridge collision i is ξ_{it_2}. So the change rate of the risk of ship-bridge collision during time (t_1,t_2) is ξ'_{it} (equation 21):

$$\xi'_{it} = \frac{\xi_{it_2} - \xi_{it_1}}{t_2 - t_1} \tag{21}$$

The ordinate of point K in the coordinate system can be obtained from equation (11), and the ordinate X_K is the warning distance of the bridge area. According to the AIS data of the ship, the speed of the ship at this time is v_i, so the time T_0 spent by the ship sailing in the early warning range of the bridge area waterway is equation (22):

$$T_0 = \frac{x_K}{v_i} \tag{22}$$

When $\xi'_{it} \cdot T_0 + \xi_{it_1} \geq 0.75$, starting the early warning function.

Specifically, the early warning means are mainly through AIS information reminder, photoelectric reminder, screen reminder, SMS reminder, etc. The whole system can also provide a support platform for the active ship-bridge collision early warning of sea-cross bridges.

3 Case analysis

For the bridges threatened by ships collision, it is of great significance to research the active early warning system for ship-bridge collision avoidance. However, even if the risk coefficient is low, it is still possible to hit against the bridge in some studies, which does not take the impact of ship's dimension on the results into account. In this paper, the risk of ship collision with bridge is calculated considering ship's dimension. If the danger range of the ship sailing on the current course is higher overall, the captain can adjust the ship's course to reduce the risk of ship-bridge collision effectively.

In order to discuss the algorithm of the risk of ship-bridge collision considering ship's dimension effectively, this paper analyzes the Zhoushan Xiushan bridge. Xiushan bridge is a sea-crossing channel connecting Daishan island and Xiushan island in Zhoushan City, Zhejiang Province, China. It is located above the sea area of the East China Sea, with a total length of 3063m, width 24m, main span of 926m, a single hole two-way span of bridge of the main navigation hole channel, and a navigation clearance width 778m, navigation clearance altitude 54.5m, and highest navigable stage 3.13m. As shown in figure 6, it is the plan of Xiushan bridge[13].

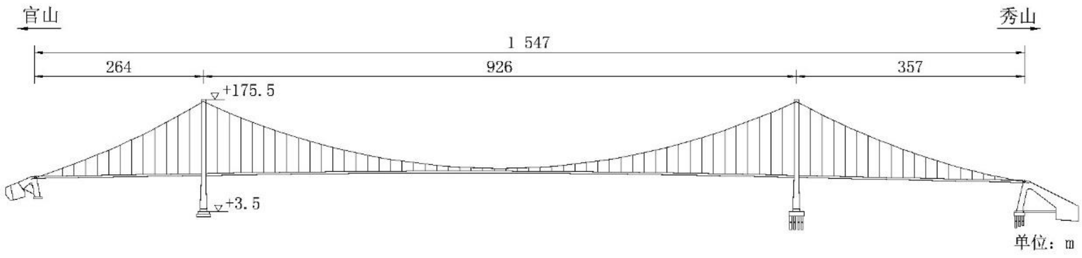

Figure 6 plan of the main navigable opening of Xiushan Bridge

The selected channel range studied in this paper is shown in table 1. A rectangular coordinate system is established, including the origin, channel direction and selection range. The longitude and latitude of the two piers of the main channel of the bridge are A(122.1869667E, 30.21495N), B(122.183511E, 30.207141N) respectively.

Table 1 Fairway range

Origin coordinates	Channel trend	Selection range
(122.2187700E, 30.212200N)	270°	122.1621900E, 30.2149500N;
		122.2065300E, 30.2149500N;
		122.2065300E, 30.2071700N;
		122.1621900E, 30.2071700N;

The "warning area" in the figure 7 is the active early warning range for ship-bridge collision avoidance in this case. The channel centerline of Xiushan bridge is taken as the longitudinal axis (x-axis). The active early warning range for ship-bridge collision's entrance direction is taken as the positive direction of Y-axis, and its' outer boundary of the entrance direction as the transverse axis (Y-axis). The intersection of the channel centerline of the bridge area and the outer boundary of the ship entrance direction is taken as the origin O (0,0), Point K is the intersection of X axis and Bridge AB in the bridge ship coordinate system.

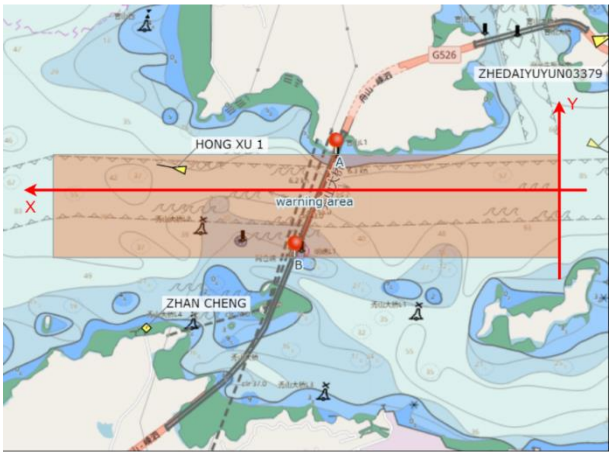

Figure 7 Establishment of the selected range coordinate system

Preprocessing the MMSI, time, captain, ship width, relative heading, speed and other data of the ship passing through the main navigation hole channel of Xiushan bridge. First, deleting the null value. Second, converting the longitude and latitude coordinates into rectangular coordinates. Third, selecting the ship data of the ship sailing forward in the I quadrant. Forth, excluding the reverse moving ship. Through the four process obtaining the forward navigation data in line with the research in this paper.

By using the Python recurrence algorithm, calling of the AIS data of Xiushan Bridge to calculate the risk of ships collision with the bridge. Part of the results are shown in table 2, which is the risk range of the ship sailing according to the current heading at a certain time point:

Table 2 ship risk range

MMSI	Time	Ship collision bridge risk range

413421830	2018/01/18 11:42:39	[0.02747, 0.04476]
413421830	2018/01/18 11:43:09	[0.50283, 0.60388]
413421830	2018/01/27 17:45:29	[0.70145, 0.81630]
413421830	2018/01/27 17:53:33	[0.01278, 0.39521]
413421830	2018/01/27 17:55:37	[0.01720, 0.26088]
413421830	2018/01/27 17:57:36	[0.01755, 0.36877]
413421830	2018/01/27 18:01:08	[0.15846, 0.27636]
413800000	2018/01/21 13:08:39	[0.00117, 0.01092]
413800000	2018/01/21 13:07:36	[0.00140, 0.00797]
413800000	2018/01/21 13:06:02	[0.00168, 0.00560]

From the table 2, we can know that at time 2018/01/27 17:45:29, the maximum ship bridge collision risk of MMSI 413421830 is 0.81630, which is higher than 0.75. That means the probability of hitting against the pier is 0.81630 if the ship sails according to the current course. At this time, the active early warning system for ship-bridge collision avoidance will activate alarm facility, telling the captain needs to change the sailing direction.

Since the Xiushan bridge is a single hole two-way span of bridge, it is only necessary to compare the maximum risk of ship collision ξ_2 with the threshold 0.75. If the bridge is a two holes one-way span of bridge, the lower limit of the risk range of ship-bridge collision will also need to consider that whether ξ_1 is less than or equal to ξ_{min0}.If ξ_1 is less than or equal to ξ_{min0} will also need to be alarmed.

4 Conclusion

In the previous study of avoidance of ship collision with bridges, ship's dimension is not taken into account enough in algorithm of risk for ship collision with bridge yet. This paper creatively proposes the algorithm of the risk of ship-bridge collision, which considering the geometric position relationship of ship, pier and channel. The algorithm considers the factors such as pier layout, navigation span, bridge and channel in the bridge area comprehensively, and putting forward an active early warning strategy according the index coming from the agorithm.

This paper illustrates the reliability of the algorithm by case study of Xiushan bridge. First, calculating the risk of ship-bridge collision of ships that passing through the main channel for sailing. Then, making a comparative analysis with the actual situation. The result shows that the calculation results of this algorithm are basically consistent with the actual ship collision risk, and can realize the prediction of ship collision risk accurately.

The algorithm considers the influence of ship's dimension on the risk of ship collision with bridge adequately. It can significantly improve the accuracy of the risk of ship-bridge collision and be applied to the active early warning system for ship-bridge avoidance collision. It is conducive to the application of the active early warning system into practice, and then heighten the level of safety including ship and bridges .

Meanwhile, the algorithm is not only suitable for bridge which has single navigable hole two-way navigation, but also suitable for bridge which has double navigable hole signle-way navigation. However, it calculates the risk of ship collision with bridge based on the ship scale in the paper justly. Maybe this algorithm can be combined with the dynamics of other ships in the next step, and realizing more accurate ship collision risk early warning and promoting the wide application of bridge active ship collision early warning system in engineering practice.

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6 Author Contributions

The authors confirm contribution to the paper as follows: study conception and design: Yihua Liu, Jingyan Zhang; data collection: Yihua Liu, Jingyan Zhang; analysis and interpretation of results: Yihua Liu, Jingyan Zhang, Bo Tu; draft manuscript preparation: Yihua Liu, Jingyan Zhang. All authors reviewed the results and approved the final version of the manuscript.

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8 References

[1] Statistical Analysis of Ship Collisions with Bridges in China Waterway[J].Journal of Marine Science and Application,2002(02): 28-32.

[2] Changhai Huang, Shenping Hu, Deyi Gao, Yongtao Xi, Shaoyong Xuan, Yu Peng. Design and implementation of dynamic risk assessment system for bridge ship collision [J]. Chinese journal of safety science, 2013, 23 (4) : 120-126. The DOI: 10.16265 / j.carol carroll nki issn1003-3033.2013.04.007.

[3] Liu, Y., & Xiao, Y. (2014). Risk Degree of Ship-bridge Collision based on Theory of Ship Collision Avoidance. International Journal of Control and Automation, 7(11), 303–316. doi:10.14257/ijca.2014.7.11.29

[4] Jun Zhong, Mingjun Liu, Chen Luo, chuhua Xie. Ship bridge collision risk assessment based on combined weight fuzzy matter-element model [J] Journal of safety and environment, 2018,18 (02): 423-428 DOI:10. 13637/j.issn. 1009-6094.2018.02.003.

[5] Keke Peng. Dynamic Ship-bridge Collision Risk Decision Method Based on Time-Dependent AASHTO Model. Journal of The Institution of Engineers (India): Series A, 102(1), 305 – 313.

[6] Tao Fu, Xiaoqian Ren, Kai Wang, Castaldo Paolo. Vessel-Bridge Collision Reliability Assessment Based on Structural Dynamic Analysis[J]. Advances in Civil Engineering, 2021,2021.

[7] Bing Wu, fan Guo, Shanshan Fu, Yang Wang, Xinping Yan. Risk early warning method of ship collision with bridge based on fuzzy logic [J]. China navigation, 2019,42 (02): 47-51 + 65.

[8] Yihua Liu. Research on ship collision risk of inland river high-grade channel bridge based on Synergetics [D]. Shanghai Maritime University, 2020.

[9] Xiangrui Yang. Research on risk assessment of ship collision with bridge based on Bayesian network [D]. Chongqing Jiaotong University, 2015.

[10] Xue Hui Yang, Xin Zhou. Research of Ship-Bridge Collision early Warning on the Basis of Fuzzy Neural Networks[J]. Advanced Materials Research,2012,2091(594-597).

[11] Jie Wang. Research on ship collision safety of bridge based on fuzzy network analysis [D]. Shanghai Jiaotong University, 2019 DOI:10. 27307/ d.cnki. gsjtu. 2019.001127.

[12] Zheng Hu, Qing Yang, Xiaonan Bu, Bo Zhang, Weijian Jiang, Xiangrui Song. Research on the conversion scheme between longitude and latitude and plane coordinates in TDOA positioning [J]. Electronic world, 2019 (03): 36-37 DOI:10.19353/j.cnki.dzsj.2019.03.013.

[13] Yuan Liu, Jiming Lin, Xiaobing Dong. Key technology of bridge tower construction of Xiushan bridge [J] World bridge, 2020,48 (05): 26-31.